Edifying The Young

by Ahmad Eid - Wednesday, April 26, 2017

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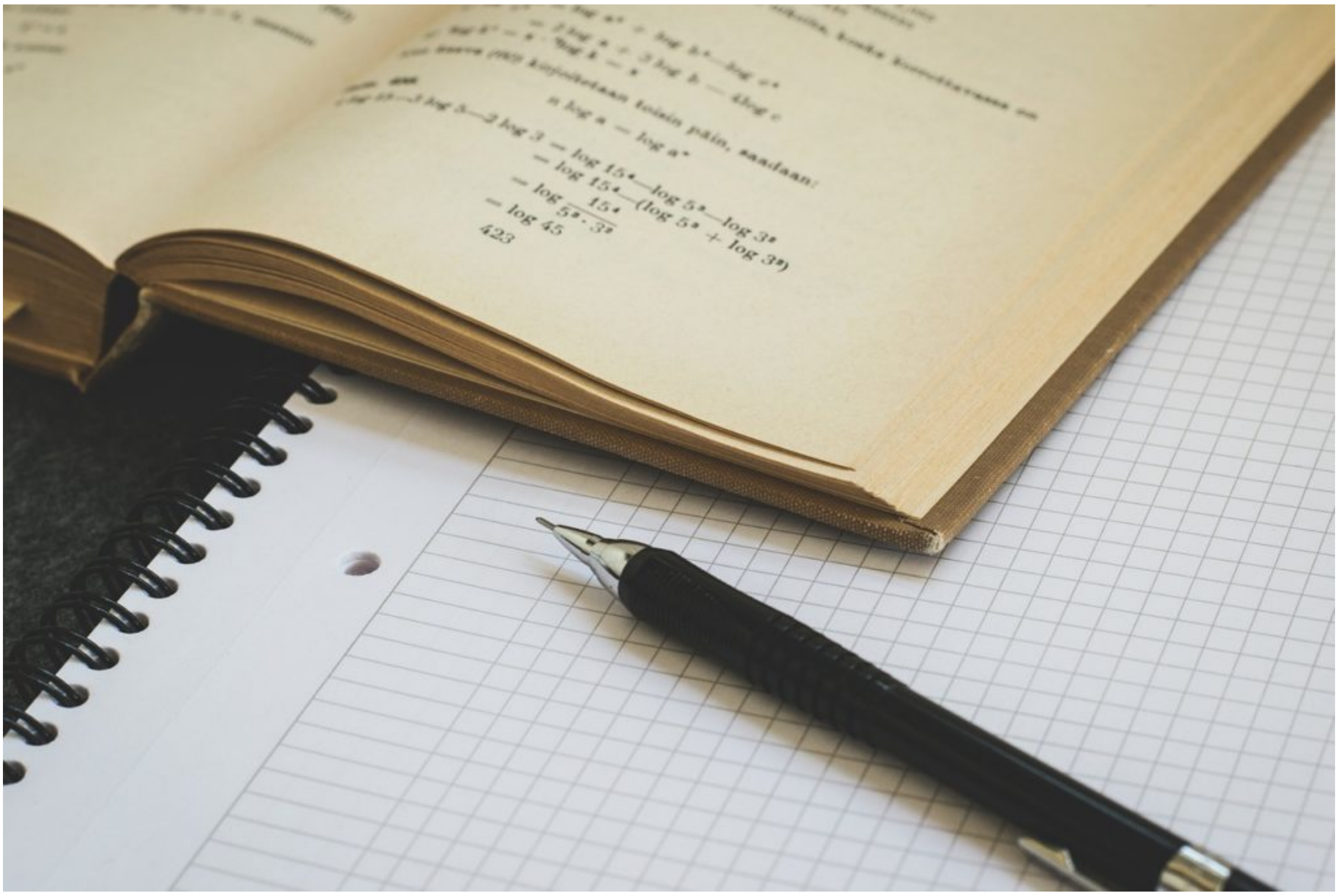

[dropcap]T[/dropcap]wo elementary courses common to modern physics, mathematics, and engineering curricula are Linear Algebra and Vector Calculus. Geometric Algebra is a natural and powerful extension of linear algebra and Geometric Calculus is even more so for vector calculus.

In this post, I interview Dr. Alan Macdonald who talks about his experience with both subjects and how he contributes in educating young undergraduate students about them through his books and online videos
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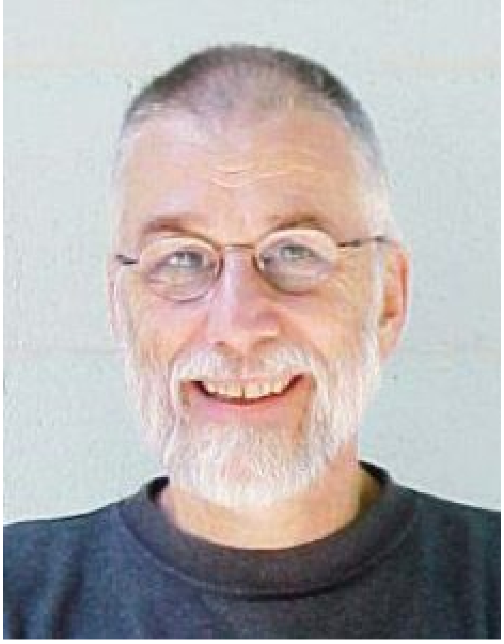

Dr. Alan Macdonald is Professor Emeritus of Mathematics at Luther College in Decorah Iowa. He received a Ph.D. in mathematics from The University of Michigan in 1970. His research interests include geometric algebra and the foundations of physics.

Dr. Alan is the author of two excellent books that introduce Geometric Algebra and Geometric Calculus to undergraduate students. In his first book ***Linear and Geometric Algebra***, he develops the first undergraduate text to cover the essentials of linear algebra and its extension to geometric algebra. Prof. David Hestenes commented on his first book saying:

> I commend Alan Macdonald for his excellent book! His exposition is clean and spare. He has done a fine job of engineering a gradual transition from standard views of linear algebra to the perspective of geometric algebra. The book is sufficiently conventional to be adopted as a textbook by an adventurous teacher without getting flack from colleagues. Yet it leads to gems of geometric algebra that are likely to delight thoughtful students and surprise even the most experienced instructors.

His second book ***Vector and Geometric Calculus*** distinguishes itself from similar books on advanced calculus by two attributes: its thoroughgoing use of Geometric Algebra and the clarity of its exposition at an undergraduate level.

Tell us about your life work in mathematics. What motivations and goals drove you through your research and teaching career? What obstacles did you face and how did you overcome them?

I have taught and published in several different areas of mathematics and physics. I faced no serious obstacles in pursuing my varied interests (except to find more than 24 hours in a day). Lucky!

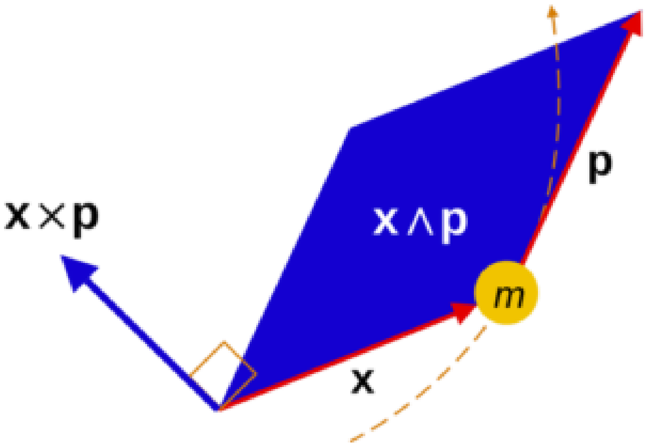
Tell us about what got you interested in Geometric Algebra. As a mathematician, what kinds of ideas in physics and geometry do you find naturally expressible using GA?

I read strong claims about geometric algebra and decided to look for myself. I was smitten. I found that geometric algebra and its extension to geometric calculus unify, simplify, and generalize vast areas of mathematics involving geometric ideas, including linear algebra, vector calculus, complex analysis, exterior algebra and calculus, tensor algebra and calculus, quaternions (3D spinors), and euclidean, non-euclidean, and projective geometries. I also found that GA provides a common mathematical language for many areas of physics (classical and quantum mechanics, electrodynamics, special and general relativity), computer science (graphics, robotics, computer vision), engineering, and other fields. People in disparate fields can now talk to each other!

Some people say "Why should we learn and use GA when we already have many mathematical tools that work?". How do you respond to this kind of question?

My response: ***Geometric algebra unifies many of your mathematical tools and works better than any of them***. Unfortunately, this has not been sufficient to induce many to switch. GA takes time to learn. And once learned, there is the problem of communicating with others who haven't learned. Nevertheless, geometric algebra is gaining adherents, slowly but surely.

Tell us about the relation between Geometric Algebra and Linear Algebra. Can the basic ideas of linear algebra be reformulated using GA? How is that? What are the benefits of such reformulations and developments?



Geometric algebra is a superset of linear algebra, with more objects, multivectors, and more operations; for example, the geometric product. So anything that linear algebra can do geometric algebra can do. Even within linear algebra, geometric algebra brings advantages. Here is a simple example. Consider the orthogonal complement of a subspace: the set of vectors orthogonal to all vectors in the subspace. For both linear and geometric algebra this ***geometric*** definition tells us what the orthogonal complement ***is***. Geometric algebra adds algebra, providing a very simple way to ***compute*** the complement (as a dual), something linear algebra can't do. This illustrates the name ***Geometric Algebra***: it does geometry algebraically.

Prof. David Hestenes talked in one of his lectures about your first undergraduates GA book and how it could have been a joint project. In what aspects does your vision for introducing GA to students relate to his?

I share David Hestenes' vision of wide acceptance of GA and the advantages this will bring.

A vital aspect of making this happen is encouraging and enabling GA's inclusion in the undergraduate mathematics curriculum. The only way that mathematics faculties will consider this is to have available GA texts written in traditional mathematics textbook style. This is my small contribution to Hestenes' vision.

Mathematicians have been infuriatingly slow in becoming interested in geometric algebra. We are repeating the short-sightedness of 120 years ago when ***Engineers welcomed Gibbs's and Heaviside's vector analysis, though the mathematicians did not*** [1. M. Kline, *Mathematical Thought from Ancient to Modern Times*, Oxford University Press, New York (1972), 791.].

What kind of software tools do you think should be used in explaining GA and GC to undergraduate students? Are there any missing features you would like to see in current tools?

I chose Alan Bromborsky's GAlgebra for my texts because it is free, it is multiplatform, it does ***symbolic*** calculations, and it is well designed. But it is far from everything I could ask for: its syntax for expressions is sometimes awkward; it requires several other programs, e.g., Python, making installation difficult for some; and it has no graphics capability.

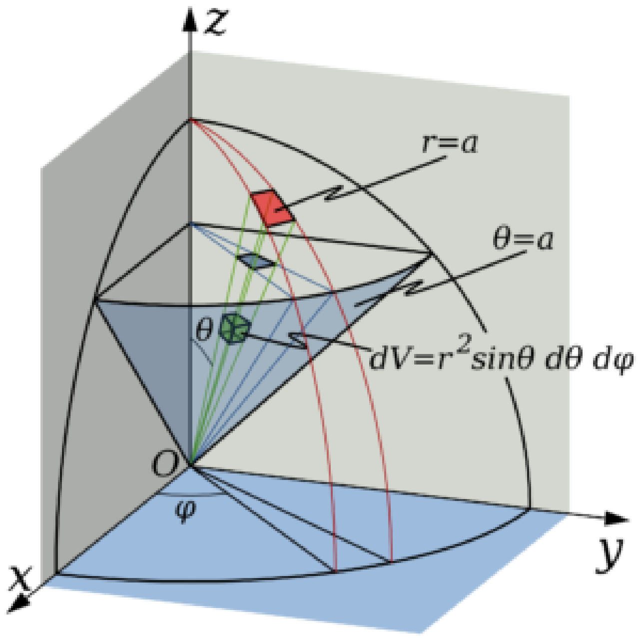
I would like to see a program with GAlgebra's strengths but not its shortcomings.

What are your expectations of the future of GA & GC in teaching?

My experience is in teaching undergraduate mathematics in the United States, so I will speak to that. Students typically take a year of scalar calculus in their first year of college, then courses in linear algebra and vector calculus in their second year.

The most natural place to introduce GA & GC is right after scalar calculus. I believe that the current second-year linear algebra and vector calculus courses should be renamed *Geometric Algebra* and *Geometric Calculus*, their purpose being to provide the basic vocabulary of mathematics in dimensions greater than one. By judiciously trimming the current courses and taking advantage of the simplifications of GA & GC, a lot of GA & GC can be covered. My two textbooks *Linear and Geometric Algebra* and *Vector and Geometric Calculus* are attempts at this.

Tell us about the mathematical developments you think are still necessary for GA & GC to take their rightful place as the primary mathematical modeling language for this century. What kind of research should be conducted to effectively relate GA & GC to mainstream mathematics and applications?



GA & GC are well developed at the elementary level, ready for use. But there are problems to be solved before they can take their rightful place.

We face a chicken and egg problem: students must learn GA & GC to apply them, and those applying vector methods, but not brought up on GA & GC, must push for their inclusion in the curriculum. And if students learn GA & GC in their second year, this cannot be a dead end: instructors in later courses must use them in their courses. Another problem: people are reluctant to publish using a mathematical formalism unknown to most. We can only do our best, making incremental progress until a critical mass is achieved.
